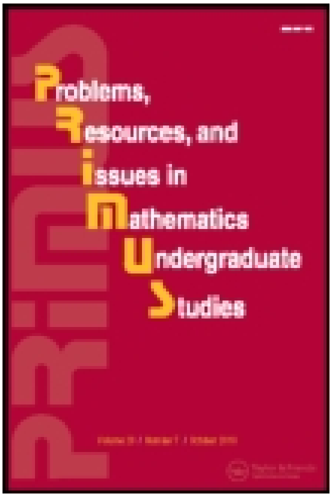

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Theron J. Hitchman & Douglas Shaw
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Tailoring Modified Moore Method Techniques to Liberal Arts Mathematics Courses

Theron J. Hitchman and Douglas Shaw

Abstract: Inquiry-based learning (IBL) techniques can be used in mathematics courses for non-majors, such as courses required for liberal arts majors to fulfill graduation requirements. Unique challenges are discussed, followed by adaptations of IBL techniques to overcome those challenges.

Keywords: Inquiry-based learning, Moore method, liberal arts.

1. INTRODUCTION

Experienced practitioners of inquiry-based learning (IBL) techniques in their mathematics classrooms often can divide their teaching careers into *before* and *after*. Many of us who are in the *after* phase feel that giving up these techniques is not even an option — no matter what challenges face us, the results that we see preclude us from ever going back to *before*. Thus, we face the challenges, and strive to improve our practice in the world of *after*. We must then question a teaching process where we are in the *after* phase for the courses we teach our majors, but we remain in *before* for the courses where we meet the majority of our students. We wish to translate our success in teaching our majors to think critically, analytically, and creatively to all of our mathematics students.

The authors are experienced in implementing IBL environments. Four years ago we felt comfortable enough with our inquiry-based practices that we decided to take up the challenge of adapting these practices for our institution's liberal arts mathematics course. We have now taught about a dozen sections of this course between us, and we feel as though we are making progress toward this goal.

Address correspondence to Theron J. Hitchman, Department of Mathematics, University of Northern Iowa, Cedar Falls, IA 50614-0506, USA. E-mail: theron.hitchman@uni.edu

Although we believe that much of what we have to share is potentially beneficial to all who are interested in inquiry-based instruction, our hope is that through this paper we can share some practical advice for those following a similar path. In our minds, the intended reader is an instructor with some experience in IBL who is moving to a new audience: the terminal mathematics course for liberal arts students. In this way we hope to add to the existing comprehensive general introductions to IBL [3].

The rest of this article is structured in the following way: we begin with a look at the possible goals for a liberal arts mathematics course. Next, we discuss some of the common challenges faced by instructors who teach such courses. We then share some advice about running these courses in the form of two lists. The first list consists of modifications to standard IBL techniques that were developed for more advanced courses. The second list is a set of suggestions about the choice of content to be covered, a decision whose importance was not appreciated by the authors when the project began. We then present, in an Appendix, some excerpts from a sample problem sequence we have used in our classes.

2. GOALS OF A LIBERAL ARTS MATHEMATICS COURSE

Colleges and universities present a variety of rationales for requiring their their liberal arts students to take a mathematics course. We urge instructors of such courses to think honestly about the question, "Given that students are going to be required to take my course, and may not ever take another mathematics course, what do I want to accomplish with our time together?" What follows is a list of possible answers to this question.

2.1. Quantitative Literacy

Citizens in our society are barraged with statements like these:

- "The latest Reuters poll shows 51% of Ohio residents are supporting Romney, and 48% are supporting Obama (with a margin of error of 3%)."
- "This home equity loan comes with a 5% interest rate and a $90 annual fee, but the fee can be waived if we go to a 5.5% interest rate."
- "A recent study has shown a correlation between potato-chip consumption and obesity, so to lose weight it is vital to cut potato-chips from your diet!"

A student who takes a liberal arts mathematics course should be able to objectively evaluate mathematical statements, and conclusions made from those statements.

2.2. Mathematical Appreciation

John Paulos [5] observed that in our society it is perfectly acceptable to loudly declare mathematical inability or disinterest, whereas nobody would dare publicly confess to illiteracy. Certainly, the public perception of mathematics as dry and dreadful affects discussions of STEM (science, technology, engineering, math) funding, advice given to high school students by their counselors, and attitudes conveyed to students by their parents. Since mathematics is a beautiful subject, it seems to betray the concept of a liberal arts education to keep this beauty hidden. We wish students to learn to appreciate art, music, and literature, should not they also leave college with an appreciation for the queen of the sciences?

2.3. Analytical Communication

Many of those who tell us how much they hate math wind up making presentations using mathematical tools. Box plots, pie charts, statistics, logical argument, interpretation of data, all of these are mathematical ideas. In a professional or personal context, there can be major consequences based on how a citizen ends the sentence, "I disagree with you because. . ." If a student successfully completes a liberal arts mathematics course, she or he should be able to use rational argument to effectively make or refute a particular case.

3. CHALLENGES

Instructors of liberal arts mathematics courses will face a variety of challenges when trying to implement an inquiry-based class structure. As with any course, one must take care to understand the goals of the course, its place in the overall curriculum, and the student audience the course serves. There are two fairly common concerns that come up that distinguish a liberal arts course from teaching a course for majors.

First, it is important to recognize the different perceptions of mathematics that students bring to a liberal arts course. In contrast with mathematics majors, these students have not usually been served well by the standard algebra-centered middle and high school curriculum. Their skills and attitude suffer for it. Instructors should not expect much in the way of fluency with symbolic algebra. Also, instructors should expect many students to arrive with misconceptions about what mathematics is and about their ability to do it. Of course, this generalization will not hold for all students, but it happens with a regularity that is not encountered in a course for mathematics majors.

Second, the population of a liberal arts math course will be much more heterogeneous in their background than a course for mathematics majors. Some students will be highly math–phobic seniors, who have put off this last obstacle between them and graduation. Some will be bright students who have done fairly well in mathematics classes in high school, without acquiring the slightest interest in the subject. Others will be students who may have a high level of mathematical ability, but did not do well in sixth grade pre-algebra, leading to later failures in algebra, trigonometry, geometry, and precalculus. Instructors are tasked with designing a course that will achieve the previously mentioned goals for as many of these students as possible.

These challenges drive the authors to some design decisions when running mathematics courses for liberal arts students. Each of the suggestions we have to share is informed by at least one of these two concerns.

4. THE BASELINE

While the Moore method of teaching mathematics is, at this point, well-defined [3], the term modified Moore method (MMM) is broader, and IBL is even more so.

Our version of a MMM class is dominated by individual student presentations in class of work done independently outside of class on a carefully constructed sequence of problems, with the rest of the class acting as a critical audience and the instructor taking on the role of classroom facilitator.

The following components comprise the core of the authors' MMM mathematics courses for mathematics majors.

1. *An efficient problem sequence*. Key concepts are developed through a sequence of problems, solved by the students with some help from the instructor, but not too much. A major part of the instructor's role is to create these sequences and add to or modify them based on class discussion. The heart of the course is the problem sequence.
2. *Student-centered classroom*. Students ask questions of the instructor and each other, make presentations, come up with conjectures, and otherwise take control of constructing mathematical knowledge. Although the instructor guides and channels the direction of inquiry, the students are given the largest amount of lattitude that is practical, given the goals of the course.
3. *Emphasis on student notes*. Since the instructor is providing definitions and problems, and the students are constructing examples and results, the textbook often becomes redundant in an MMM course. Students are expected to make their own course notes that replace much of the functionality of a textbook.

Overall, the stress in such a course is for students to independently construct knowledge, and then defend their constructions in front of a jury of their peers and the instructor. There is obviously more that can be said about these courses, but the above is the starting point from which we adapt.

5. SOME SUGGESTED ADAPTATIONS OF MMM PEDAGOGY

As we have worked on adapting inquiry-based methods to our liberal arts mathematics course, we have collected some ideas on what can help make the experience more successful. Again, these are written from a perspective of instructors who are comfortable running inquiry-based classes for mathematics majors, and so these ideas should be read as adaptations of a MMM.

5.1. Structural Differences

Although we appreciate the “think for yourself, and stand up for yourself” philosophy behind the MMM, we find that this makes for too strict an environment for our liberal arts courses full of nonmajors. To take the edge off of the situation, we use a few simple modifications.

1. We use group work liberally. Groups of three or four work best [6]. Many of our students are familiar with this mode of work from high school. It is important to mix the groups often. Too much familiarity in a group can lead to issues with staying on task. We also find it important to instruct the students to introduce themselves each time they form a group. We use methods adapted from Ken Brufee [1], but there are plenty of excellent methods out there.
2. We begin every class session having a student “break the ice,” to lessen the fright associated with presenting an idea to the whole class. We have found it best to do this by having a student volunteer to stand up, introduce himself or herself, and tell a joke that would be appropriate for an elementary school audience. If nobody volunteers, we wait until somebody does–it rarely takes more than 60 seconds for this to happen. The jokes are terrible, everyone laughs, and then the ice is broken. There are other icebreakers, of course.
3. Letting students work through misconceptions and find their own understanding is still important. However, there are times when a gentle course correction is appropriate. We find it helpful to interrupt the whole class with hints and tips, rather than keep it to just one group of students at a time. It is often the case that some basic misunderstanding between the instructor and one student actually lies between the instructor and nearly the whole class. Thus, when we find an opportunity to redirect one particular group with a helpful question or a pointed remark, we announce it to the whole class. The point of this is not to save time, though it can have that effect. More importantly, we find that our liberal arts students can lack the ability to self-correct and re-engage when stuck. Making a class-wide announcement can minimize the amount of unproductive frustration. (If the day’s work calls for some productive frustration, that is another matter.) See also [4].

In our liberal arts mathematics courses, class time is mostly students doing mathematics in small groups, with occasional interruptions for student presentations to the whole class. The role of the instructor is to provide guidance when necessary, playing coach, cheerleader, mentor, and classroom manager.

5.2. Redundancy

Ideas and techniques should be repeated more often and over a longer period of time than for a majors course. Our liberal arts students need more time to recognize and absorb the main ideas lying behind the problems we ask them to solve. This is especially true of some of the deeper-lying mathematical culture. In particular, students should have plenty of opportunity to practice using language with the precision shown by a mathematician.

We find that this point is especially important when it comes to the design of a problem sequence. For example, many instructors take satisfaction in writing a concise problem sequence. This may be fine for students who have absorbed the culture of mathematical work and can spot important ideas and techniques when they arise, but it does not work well for students in our liberal arts course. Using several problems that require the same idea, or very similar ideas, is crucial for getting more understanding in less time.

One good example of the redundancy principle is this trick: have students work in groups on a problem set during one particular meeting, and adjourn when students are only "half finished." Then start the next meeting by dividing the students into new groups and have them work on the same set of problems, redoing their conversations from the beginning. We have seen this be effective for conveying deep ideas and for teaching the students to speak mathematically with their peers.

5.3. Sign Posting

A simple way to help a more general audience keep up with the sophistication of an inquiry-based course is to pay careful attention to the structure of the class and handouts.

1. Frame each class meeting. At the beginning of each day, spend two minutes reminding the students what they have accomplished so far and what questions remain. At the end of the session, spend another two minutes recapping the highlights of their achievements and then hint at an interesting question that will come next. As the semester goes on, it helps to switch to asking the students to perform these tasks. This trains them to try to understand context on their own.

2. Headline individual main ideas when they occur up. If your sequence of activities and questions has some built-in redundancy, all the better. This will help students to see which things are "big ideas," and which are less important. Again, the instructor need not do this explicitly. One can stick to the spirit of IBL by asking simple key questions—it helps students recognize the importance of their answer if that important question recurs.
3. Use careful typesetting as visual cues to the structure of a problem sequence. Although we do not advocate blue boxes containing important theorems, we still find it helpful to think about how problems are displayed. We are deliberate about setting out the main ideas with a visual cue, such as boldface. If Question 12 is where the main ideas come together, then it is important to point this out in the course notes with a visual cue. Moreover, when doing this, it is important to choose a consistent way of doing it throughout course materials.

5.4. Immediate Feedback

We find it imperative to give students feedback as quickly as possible. Our liberal arts students do not naturally re-evaluate their thinking between class meetings, so a misconception left unaddressed at the end of a session can become received wisdom by the next one. It is not to say that the instructor should impose his or her viewpoint upon the students because the hour is ending. Rather, it should somehow be made clear before the meeting breaks up that the ideas in question are not yet convincing and still need proper attention and evaluation.

5.5. Meta-Instruction

Many of our liberal arts students have missed subtle, but important, bits of mathematical culture. To remedy this, we give explicit instruction on how to study mathematics, how to do mathematical work, and effective methods of problem solving. In fact, it also seems important to point out that we are giving such instruction as we are doing it. Students new to IBL environments can feel left alone, and our liberal arts students often have negative views of their abilities. By pointing out that we are giving instruction in how mathematics is done, we can avoid some feelings of abandonment.

Periodically, we discuss the structure of the course itself with the students, in the spirit of "back-and-forth" as opposed to "feedback–response." In this way, we are reinforcing the major idea of increased student responsibility for their learning. The students gain motivation as they feel ownership of their class, and their suggestions often lead to improvement.

An important aspect of meta-instruction lies in how we deal with modeling correct work. Our liberal arts mathematics students display less confidence in the quality of their own arguments than do our majors. Although we do not wish to act as the arbiter of correctness (we would much rather leave that to the students), the students need some indication that they have made a positive contribution to help them build confidence. Thus, we are very careful to point out that when the class is convinced, then the argument is deemed correct. Again, we point out that we are sharing with them the way mathematicians actually decide on the value of things. "Is everyone convinced? Then that is what a proof looks like." This is a strong set of signals: the students have the instructor's trust, the students have the responsibility for deciding for themselves what to believe, and the instructor has just stated (or reiterated) what his or her expectations are for student work.

We find that students in our liberal arts mathematics course often want, and need, some instruction on how to study. Given that the structure of the course is likely new to them, this is not unusual or unwarranted. As one feature of the course will be to introduce the habits of mind of working mathematicians, it is important to share with the students what kinds of things the instructor feels are valuable for self-improvement and for mastering the new ideas. And again, we find it more effective to point out what is going on while it is happening. We do not just say, "This is how mathematicians work and learn." We say, "This is how mathematicians work and learn, and this is what you should do. It will help you improve, and it will help you prepare for exams."

6. SOME SUGGESTIONS ABOUT CHOOSING MATERIAL

We find that an important factor in the level of success of our mathematics for liberal arts courses is the selection of mathematical topics to be studied. Appropriate choice of content will vary from place to place depending on course aims and needs, but we have some general principles we use in determining what mathematics the students will try to develop during the course. We also share some ideas for topics we have taught to a liberal arts audience.

6.1. Choosing Content

Our principles for choosing content are the following:

1. *Start fresh*. It is important to begin with a topic that is new to as many students as possible. This has a nice leveling effect, putting the students all at the same starting place, with no one having a perceived advantage of experience. This also allows the students to reset their relationship with

mathematics. When the ideas are clearly new, there is an allowance for working on the basics without guilt.

2. *Cover the basics*. Great care is necessary in introducing new topics. In our experience, students in liberal arts courses need more opportunity to talk through basic issues, especially those related to the careful use of language. It can help the course run more smoothly if the topics under consideration make explicit allowance for working on foundation concepts.
3. *Ground the content*. In order to engage the students, it is important to pick things they will view as worthy of discussion. Three good characteristics are:

   (a) the mathematics connects with something they already know;
   (b) the mathematics is amazingly cool (from the audience's perspective);
   (c) the mathematics is self-evidently practical and useful.

   It is not necessary to have all three characteristics for each topic, but two properties are better than one. If a topic does not satisfy any of these, it may be best to reconsider its use.
4. *High ceiling*. The best topics are grounded in something the students know, so they can get started right away working on the basics. However, it is also important that the topic has some real depth. The end of a unit should be wonderful, perhaps surprising or mind-bending, and should encourage further investigation. This helps connect students to mathematics as a subject: they have learned something significant, and they understand that it is still not the end of the story. To amplify this point, we like to show the students at least one unanswered question to demonstrate that mathematics is a living subject.
5. *Novelty*. At least some of the topics you choose should be new to the students. If the students think they know the content already, you will easily get trouble with engagement. We find this plagues our attempts to teach some ideas from probability and statistics, since these topics are becoming more prevalent in the high school curriculum.
6. *Avoid heavily algebraic material*. The students have likely spent years not mastering high school algebra. Another rehash, no matter how well executed, is unlikely to succeed.
7. *Spread it around*. Choose units that are completely independent rather than a sequence that builds. We find that for a 14 week semester, three topics make for a satisfying class. Each topic can be developed over about a month (12 meetings or so). This gives ample time for the students to get acclimated to a new set of ideas and make enough progress to do something interesting. From the other direction, it is useful when the material gets more challenging to be able to truthfully tell the students that classes will not be so abstract forever. The prospect of letting very challenging material go soon in favor of a new topic which will start from a much simpler set of questions can be enough to encourage students to dig in for a conclusion. A few radical

changes in direction can provide some relief to the fatigued. Also, since not every topic will excite every student, a selection of unrelated topics provides multiple chances to change a student's mind about mathematics.

6.2. Some Topics We Have Tried

Here is a selection of topics we have tried with varying degrees of success.

1. Countable and uncountable infinities (see Appendix).
2. The classification of compact surfaces with boundary.
3. Continued fractions.
4. The classification of wallpaper patterns and friezes.
5. Voting theory.
6. Basic statistics and confidence intervals up to polling.
7. Conditional probability and Bayesian reasoning.

7. CONCLUSIONS

Instructors finding success teaching MMM courses to their majors may feel that teaching their liberal arts courses in a more traditional format is taking a step backwards. With some relatively simple adaptations, we find that the successes of the IBL format in a majors course can be replicated in the liberal arts context. Students can leave their liberal arts math class knowing that they have done much more than checking off a requirement; they can exit knowing that they have gained quantitative literacy, a new appreciation of mathematics in general, and an increased ability to communicate analytically. We advocate four major types of adaptations relative to courses for majors.

1. *Learning Support*. As much as possible, provide immediate feedback and meta-instruction.
2. *Meeting structure*. Increase group work, icebreakers, and instructor-provided guidance.
3. *Problem sequences*. Increase signposting and redundancy.
4. *Course content*. Start with something new to most of the students, and then proceed to topics that are cool, self–evidently practical, and connected to previous knowledge. Remember that the field of mathematics is broad enough and beautiful enough to contain plenty of topics that do not rely heavily on algebraic manipulation.

One of the great joys of teaching is helping students realize powers and abilities that they did not know they possessed. It stands to reason that the greatest potential to experience this joy is in a class of liberal arts students, many of whom have been bruised mathematically, and have no idea what they can accomplish. Also, delightfully, it is often the case that we as instructors have teaching powers and abilities that we did not know we possessed. We encourage instructors to extend their IBL teaching to this population, and to share their experiences.

APPENDIX

Samples from a Liberal Arts Problem Sequence

The following is a sample from a problem sequence we have used on the nature of countably and uncountably infinite sets. This was the first topic covered in a liberal arts mathematics course. We have retained the original numbering to give a sense of the pacing we find appropriate for our liberal arts course.

Comparing Large Sets

Task 1 *Let's explore the idea of a one-to-one correspondence.*

- *Is there a one-to-one correspondence between the people in this classroom and the spleens in this classroom?*
- *Every student at a certain college is assigned to a dorm room. Does this imply that there is a one-to-one correspondence between dorm rooms and students?*
- *Every child has one and only one birth mother. Is there a one-to-one correspondence between the set of all children and the set of all birth mothers?*
- *Is there a one-to-one correspondence between U.S. residents and their social security numbers?*

Task 3 [2] *The following are two collections of the symbols @ and &:*

@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@

@@@@@@@@@@@@@@@@

&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&

&&&&&&&&&&&&&&&&

Are there more &s than @s? Or the same amount? How do you know?

Task 7 *Let S be the set of 8 digit numbers, such as 72356790 and let T be the set of 8 letter words that can only use the the letters A through J and do not begin with the letter A.*

(a) *Write out two elements of S.*
(b) *Write out two elements of T.*
(c) *Prove that S and T have the same size.*

Task 11 *Let* $\mathbb{N}$ *be the set of natural numbers:* $\{1,2,3,...\}$. *Let* $2\mathbb{N}$ *be the set of even natural numbers* $\{2,4,6,...\}$. *Prove that* $\mathbb{N}$ *and* $2\mathbb{N}$ *have the same size. It does not suffice to say "infinity equals infinity." Think of how you have done similar tasks.*

Task 13 *Let* $\mathbb{N}$ *be the set of natural numbers. Let W be the set of all words. Which is larger,* $\mathbb{N}$ *or W? Or are they the same size?*

Task 21 *Let S be the set of ways to line up 8 / symbols and 3 * symbols in a row. For example, these are different elements of S:*
///*//*//*/ //*///*//*/ */////**///

Let T be the set of ways to write 8 as the ordered sum of 4 whole numbers. For example, these are different elements of T:

$3+2+2+1=8$ $8+0+0+0=8$
$2+3+2+1=8$ $0+8+0+0=8$
$0+5+0+3=8$ $2+2+2+2=8$

(*Notice that, unlike last time, we are going to consider different orderings as different things.* $8+0+0+0=8$ *and* $0+8+0+0=8$ *are not the same element of T.*)

Prove that S and T have the same size.

Task 22 *Look at this table of fractions!*

1/1	2/1	3/1	4/1	5/1	6/1	...
1/2	2/2	3/2	4/2	5/2	6/2	...
1/3	2/3	3/3	4/3	5/3	6/3	...
1/4	2/4	3/4	4/4	5/4	6/4	...
1/5	2/5	3/5	4/5	5/5	6/5	...
1/6	2/6	3/6	4/6	5/6	6/6	...
⋮	⋮	⋮	⋮	⋮	⋮	⋱

Prove that every element of $\mathbb{Q}^+$ *appears at least once in the table.*

Task 23 *Look at this list of numbers!*

$$\frac{1}{1},\frac{2}{1},\frac{1}{2},\frac{3}{1},\frac{2}{2},\frac{1}{3},\frac{4}{1},\frac{3}{2},\frac{2}{3},\frac{1}{4},\frac{5}{1},\frac{4}{2},\frac{3}{3},\frac{2}{4},\frac{1}{5},\frac{6}{1},\ldots$$

Use the result of the previous problem to show that every element of $\mathbb{Q}^+$ *apears at least once in the list.*

Task 24 *Prove that* $\mathbb{N}$ *and* $\mathbb{Q}^+$ *have the same size.*

Task 25 *Is the result from the last task amazing or what?*

Task 33 *Let I (for "interval") be the set of real numbers between zero and one. We begin by trying to show that I is countably infinite. Don't worry, I am not going to ask you to come up with a one-to-one correspondence between the elements of I and the elements of* $\mathbb{N}$! *What we will do is pretend we have one, and see what that might tell us. Suppose that such a scheme starts like this:*

$$\begin{array}{l} 1 \leftrightarrow 0.1235542346234\ldots \\ 2 \leftrightarrow 0.1435542346234\ldots \\ 3 \leftrightarrow 0.2335247700809\ldots \\ 4 \leftrightarrow 0.7878709870777\ldots \\ 5 \leftrightarrow 0.1010000010011\ldots \\ 6 \leftrightarrow 0.0000030000040\ldots \\ \vdots \quad \vdots \quad \ddots \end{array}$$

We make a new element of I, called A, by specifying its decimal representation. Here is how: to get the nth digit of A, look at the nth digit past the decimal point of the nth item in our list. If that digit is a 3*, we make the nth digit of A a* 7*; if that digit is not a* 3*, then we make the nth digit of A a* 3*.*

Write out the first six digits of A.

Task 34 *Show that A cannot appear on our list.*

Task 35 *Use the previous idea to prove that I is not a countably infinite set! Your proof might begin "Assume that I was countable. Then we would be able to . . ."*

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BIOGRAPHICAL SKETCHES

Theron J. Hitchman is an Associate Professor of Mathematics at the University of Northern Iowa. He received his Ph.D. from the University of Michigan in 2003. His professional interests include geometry, dynamical systems, inquiry-based instruction, and undergraduate research. He is a Project NExT Fellow (Sun dot), and is active in the MAA. In his spare time he plays and coaches soccer, and tells ridiculous stories to his children.

Douglas Shaw is a Professor of Mathematics at the University of Northern Iowa. He received his Ph.D. from the University of Michigan in 1995, way before Dr Hitchman achieved his doctorate. His professional interests include the Collatz Conjecture, tertiary mathematics education, and ways of exposing K-12 students to important unsolved mathematics problems. He also coaches improv, and is working on a book about rhetorical figures.